**The shape of the head of the spear**

The Important formula is **Cayley-menger determinant**

This formula calculates the volume of a solid figure using the coordinates of its vertices in the coordinate system. Just assume that this solid has n+1 vertices, and means the distance between vertices a and b.

In our problem, we need to find the volume and cross-section area of the spear. We could just assume that its shape is like a pyramid.

O (0, 0, 0)

D (a, -b, -c)

C (a, -b, c)

A (a, b, c)

B (a, b, -c)

As the graph shows above, if the shape of the spear is a pyramid, it has four vertices. So, the n in the Cayley-Menger determinant should be 4. Then, we could use the determinant to find out the volume of this spear.

However, the shape of the spear does not need to be like the graph above; the shape of the bottom could be any polygon.

And the cross-section areas are easy to find too.

Let`s assume the vertical cross -section looks like the graph below

C (a, -b, c)

D (a, -b, -c)

B (a, b, -c)

A (a, b, c)

So, the vertical cross-section area is

And let`s assume the horizontal cross-section area is

H=a

S=2c

So, the volume of horizontal cross-section area is

And the mass of the head of the spear is

All the calculations above are based on the assumption that the shape of the head of the spear is a pyramid, so it`s just a special case. However, the coordinates of the vertices on the bottom of the head could be changed. Thus, the volume, mass, and cross-section areas will change, too.

For topic C, I think we could just assume that the bottom of the head of the spear is a polygon (An easier way is just assuming that the bottom is a rectangle) perpendicular to the Y-axis and symmetric to the center of the Y-axis. Because this could make the question much easier. Based on my knowledge, maybe we can use three nested loop to find out the volume of the spear, vertical cross-section area, horizontal cross-section area when the x coordinates, y coordinates, and z coordinates vary from 0 to a certain maximum value.